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The Categories Defended

MS 308. [Published in CP 5.66–81, 88–92 (in part) and in HL 167–88. This is the third Harvard lecture, delivered on 9 April 1903.] In this lecture Peirce goes into more detail concerning the nature of his categories and uses them to distinguish three kinds of signs: icons, indices, and symbols. He analyzes in particular one type of symbol, the proposition, which always refers to its object in two ways: indexically, by means of its subject, and iconically, by means of its predicate. Peirce defends his categories against the view he attributes to A. B. Kempe that Thirdness is not required to express the relations of mathematics, and he argues for the independence of Firstness, Secondness, and Thirdness.

I

Ladies and Gentlemen:

Category the First is the Idea of that which is such as it is regardless of anything else. That is to say, it is a *Quality* of Feeling.

Category the Second is the Idea of that which is such as it is as being Second to some First, regardless of anything else and in particular regardless of any *law*, although it may conform to a law. That is to say, it is *Reaction* as an element of the Phenomenon.

Category the Third is the Idea of that which is such as it is as being a Third, or Medium, between a Second and its First. That is to say, it is *Representation* as an element of the Phenomenon.

A mere complication of Category the Third, involving no idea essentially different, will give the idea of something which is such as it is by virtue of its relations to any multitude, enumerable, denumeral, or abnumerable[1] or even to any supermultitude of correlates; so that this Category suffices of itself to give the conception of True Continuity, than which no conception yet discovered is higher.

Category the First, owing to its Extremely Rudimentary character, is not susceptible of any degenerate or weakened modification.

Category the Second has a *Degenerate* Form, in which there is Secondness indeed, but a weak or Secondary Secondness that is not in the pair in its own quality, but belongs to it only in a certain respect. Moreover, this degeneracy need not be absolute but may be only approximative. Thus a genus characterized by Reaction will by the determination of its essential character split into two species, one a species where the secondness is strong, the other a species

where the secondness is weak, and the strong species will subdivide into two that will be similarly related, without any corresponding subdivision of the weak species. For example, Psychological Reaction splits into Willing, where the Secondness is strong, and Sensation, where it is weak; and Willing again subdivides into Active Willing and Inhibitive Willing, to which last dichotomy nothing in Sensation corresponds. But it must be confessed that subdivision, as such, involves something more than the second category.

Category the Third exhibits two different ways of Degeneracy, where the irreducible idea of Plurality, as distinguished from Duality, is present indeed but in maimed conditions. The First degree of Degeneracy is found in an Irrational Plurality which, as it exists, in contradistinction from the form of its representation, is a mere complication of duality. We have just had an example of this in the idea of Subdivision. In pure Secondness, the reacting correlates are, as I showed in the last lecture, *Singulars*, and as such are *Individuals*, not capable of further division.[2] Consequently, the conception of Subdivision, say by repeated dichotomy, certainly involves a sort of Thirdness, but it is a thirdness that is conceived to consist in a second secondness.

The most degenerate Thirdness is where we conceive a mere Quality of Feeling, or Firstness, to represent itself to itself as Representation. Such, for example, would be Pure Self-Consciousness, which might be roughly described as a mere feeling that has a dark instinct of being a germ of thought. This sounds nonsensical, I grant. Yet something can be done toward rendering it comprehensible.

I remember a lady's averring that her father had heard a minister, of what complexion she did not say, open a prayer as follows: "O Thou, All-sufficient, Self-sufficient, Insufficient God." Now pure Self-consciousness is Self-sufficient, and if it is also regarded as All-sufficient, it would seem to follow that it must be Insufficient. I ought to apologize for introducing such buffoonery into serious lectures.[3] I do so because I seriously believe that a bit of fun helps thought and tends to keep it pragmatical.

Imagine that upon the soil of a country that has a single boundary line, thus ⊜ and not ⊜ ⊜ or ⊜, there lies a map of that same country. This map may distort the different provinces of the country to any extent.

But I shall suppose that it represents every part of the country that has a single boundary by a part of the map that has a single boundary; that every part is represented as bounded by such parts as it really is bounded by, that every point of the country is represented by a single point of the map, and that every point of the map represents a single point in the country. Let us further suppose that this map is infinitely minute in its representation so that there is no speck on[4] any grain of sand in the country that could not be seen represented upon the map if we were to examine it under a sufficiently high magnifying power. Since, then, everything on the soil of the country is shown on the map, and since the map lies on the soil of the country, the map itself will be portrayed in the map, and in this map of the map everything on the soil of the country can be discerned, including the map itself with the map of

the map within its boundary. Thus there will be within the map a map of the map, and within that a map of the map of the map and so on *ad infinitum*. These maps being each within the preceding ones of the series, there will be a point contained in all of them, and this will be the map of itself. Each map which directly or indirectly represents the country is itself mapped, in the next, that is, is in the next, is represented to be a map of the country. In other words each map is *interpreted* as such in the next. We may therefore say that each is a representation of the country *to* the next map; and that point that is in all the maps is in itself the representation of nothing but itself and to nothing but it itself. It is therefore the precise analogue of pure self-consciousness. As such it is *self-sufficient*. It is saved from being insufficient, that is, as no representation at all, by the circumstance that it is not *all-sufficient*, that is, is not a complete representation but is only a point upon a continuous map.

I dare say you may have heard something like this before from Professor Royce; but if so, you will remark an important divergency.[5] The idea itself belongs neither to him nor to me, and was used by me in this connection thirty years ago.[6]

The relatively degenerate forms of the Third category do not fall into a catena, like those of the Second. What we find is this. Taking any class in whose essential idea the predominant element is Thirdness, or Representation, the self-development of that essential idea,—which development, let me say, is not to be compassed by any amount of mere "hard thinking," but only by an elaborate process founded upon experience and reason combined,— results in a *trichotomy* giving rise to three subclasses, or genera, involving respectively a relatively genuine thirdness, a relatively reactional thirdness or thirdness of the lesser degree of degeneracy, and a relatively qualitative thirdness or thirdness of the last degeneracy [see figure 1].[7] This last may subdi-

Fig. 1

vide, and its species may even be governed by the three categories, but it will not subdivide in the manner which we are considering by the essential determinations of its conception. The genus corresponding to the lesser degree of degeneracy, the reactionally degenerate genus, will subdivide after the manner of the second category forming a catena, while the genus of relatively genuine Thirdness will subdivide by trichotomy just like that from which it resulted. Only as the division proceeds, the subdivisions become harder and

harder to discern. The representamen, for example, divides by trichotomy into the general sign, or *symbol*, the *index*, and the *icon*. An *icon* is a representamen which fulfills the function of a representamen by virtue of a character which it possesses in itself, and would possess just the same though its object did not exist. Thus, the statue of a centaur is not, it is true, a representamen if there be no such thing as a centaur. Still, if it represents a centaur, it is by virtue of its shape; and this shape it will have, just as much, whether there be a centaur or not. An *index* is a representamen which fulfills the function of a representamen by virtue of a character which it could not have if its object did not exist, but which it will continue to have just the same whether it be interpreted as a representamen or not. For instance, an old-fashioned hygrometer is an *index*. For it is so contrived as to have a physical reaction with dryness and moisture in the air, so that the little man will come out if it is wet, and this would happen just the same if the use of the instrument should be entirely forgotten, so that it ceased actually to convey any information. A *symbol* is a representamen which fulfills its function regardless of any similarity or analogy with its object and equally regardless of any *factual* connection therewith, but solely and simply because it will be interpreted to be a representamen. Such for example is any general word, sentence, or book.

Of these three genera of representamens the *Icon* is the Qualitatively Degenerate, the *Index* the Reactionally degenerate, while the *Symbol* is the relatively genuine genus.

Now the *Icon* may undoubtedly be divided according to the categories but the mere completeness of the notion of the *icon* does not imperatively call for any such division. For a pure icon does not draw any distinction between itself and its object. It represents whatever it may represent, and, whatever it is like, it in so far is. It is an affair of suchness only.

It is quite otherwise with the *Index*. Here is a reactional sign, which is such by virtue of a real connection with its object. Then the question arises, is this dual character in the index so that it has two elements, by virtue of the one serving as a substitute for the particular object it does, while the other is an involved *icon* that represents the representamen itself regarded as a quality of the object,—or is there really no such dual character in the index, so that it merely denotes whatever object it happens to be really connected with just as the icon represents whatever object it happens really to resemble? Of the former, the relatively genuine form of index, the hygrometer is an example. Its connection with the weather is dualistic, so that by an involved *icon*, it actually conveys information. On the other hand any mere landmark by which a particular thing may be recognized because it is as a matter of fact associated with that thing, a proper name without signification, a pointing finger, is a degenerate index. Horatio Greenough, who designed Bunker Hill Monument, tells us in his book that he meant it to say simply "Here!" It just stands on that ground and plainly is not movable. So if we are looking for the battlefield, it will tell us whither to direct our steps.[8]

The *Symbol*, or relatively genuine form of Representamen, divides by trichotomy into the Term, the Proposition, and the Argument. The term corresponds to the icon and to the degenerate index. It does excite an icon in the imagination. The proposition conveys definite information like the genuine index, by having two parts of which the function of the one is to indicate the object meant while that of the other is to represent the representamen by exciting an icon of its quality. The argument is a representamen which does not leave the interpretant to be determined as it may by the person to whom the symbol is addressed; but separately represents what is the interpreting representation that it is intended to determine. This interpreting representation is, of course, the conclusion. It would be interesting to push these illustrations further; but I can linger nowhere. As soon as a subject begins to be interesting I am obliged to pass on to another.

II⁹

The three categories furnish an artificial classification of all possible systems of metaphysics which is certainly not without its utility. The scheme is shown in figure 2. It depends upon what ones of the three categories each system admits as important metaphysico-cosmical elements.
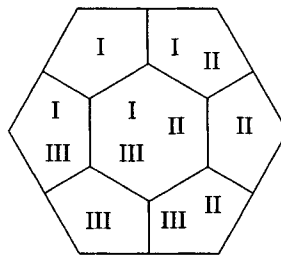

Fig. 2

One very naturally and properly endeavors to give an account of the universe with the fewest and simplest possible categories: *Praedicamenta non sunt multiplicanda praeter necessitatem.* We ought therefore to admire and extol the efforts of Condillac[10] and the Associationalists to explain everything by means of qualities of feeling.

If, however, this turns out to be a failure, the next most admirable hypothesis is that of the corpuscularians, Helmholtz and the like, who would like to explain everything by means of mechanical force, which they do not distinguish from individual reaction. That again failing, the doctrine of Hegel, who regards Category the Third as the only true one, is to be commended. For in the Hegelian system the other two are only introduced in order to be *aufgehoben.* All the categories of Hegel's list from Pure Being up appear to me very manifestly to involve Thirdness, although he does not appear to recognize it, so immersed is he in this category. All three of those simplest systems having worked themselves out into absurdity, it is natural next in accordance with the

maxim of Parsimony[11] to try explanations of the Universe based on the recognition of two only of the Categories. The more moderate nominalists who nevertheless apply the epithet *mere* to thought and to representamens may be said to admit Categories First and Second and to deny the Third. The Berkeleyans for whom there are but two kinds of entities,—souls, or centers of determinable thought, and ideas in the souls, these ideas being regarded as pure statical entities, little or nothing else than Qualities of Feeling,—seem to admit Categories First and Third and to deny Secondness, which they wish to replace by Divine Creative Influence, which certainly has all the flavor of Thirdness. So far as one can make out any intelligible aim in that singular hodgepodge, the Cartesian metaphysics, it seems to have been to admit Categories Second and Third as fundamental and to deny the First. Otherwise, I do not know to whom we can attribute this opinion which certainly does not seem to be less acceptable and attractive than several others. But there are other philosophies which seem to do full justice to Categories Second and Third and to minimize the First, and among these perhaps Spinoza and Kant are to be included.

III

I desire in the first place to defend the three Categories as the three irreducible and only constituents of thought, leaving aside for the present the question of the parts they play in the economy of the Universe.

In regard to the First, however, I trust I said enough in the last lecture. I will not return to that.

As to Category the Second, if I were asked to say of what indisputable advantage to philosophy the exact study of the logic of relations had been, and if in answering the question I considered only the manner in which it presents itself to my own mind, I should unhesitatingly mention, as its first and most unquestionable service, that it had put, in the minds of every student of it, the Category of Reaction entirely beyond all doubt as an irreducible element of thought.

And yet the lamented Schröder, in the introduction to his first volume,[1] written, it is true, as is evident enough throughout that and the second volume, before he had very thoroughly studied the logic of relations, appears to me, although probably without fully perceiving the bearing of his doctrine, to take ground quite inconsistent with such recognition of Category the Second.[12] I shall seize this opportunity to enter my protest against the position to which I allude, that position being in my opinion fatal to any sound pragmatistic conceptions. In that introduction Schröder proclaims himself a follower of Sigwart[13] in regard to the fundamentals of logic; and expressly says that he dissents from my opinions because of the reasons that Sigwart has given. I entertain a relatively high respect for Sigwart, such respect as I entertain for Rollin as a historian, Buffon as a naturalist, Priestley as a chemist, and Biot as a physicist.[14] I would go so far as to pronounce him one of the most critical

and exact of the inexact logicians. The particular point now in question is this. I had said that the question of whether a reasoning is sound or not is purely a question of *fact;* namely the fact whether or not such premisses as those of whatever argument might be under criticism could be true while the conclusion was false, in case it was proposed as a necessary reasoning,— and in case of probable reasoning *[there is]* some analogous question of *fact* corresponding to the pretensions of the argument. I thus, you will *[have]* perceived, referred the matter to the Category of Reaction, to which the conceptions of existence and fact chiefly belong. But Schröder dissents from this because Sigwart has said that the question whether a given inference is logical or not must in the last resort come down to a question of how we feel,—a question of logical *Gefühl,* to use his own expression, which is to refer truth to the category of Quality of Feeling. This he undertakes to demonstrate. For, he says, if any other criterion be employed, the correctness of this criterion has to be established by reasoning; and in this reasoning that is thus antecedent to the establishment of any rational criterion there is nothing else upon which we can rely but *Gefühl,* so that *Gefühl* is that to which any other criterion must make its ultimate appeal as its ground. Good, say I. This is the sort of reasoning that advances philosophy,—a good square, explicit, and tangible fallacy, that can be squarely met and definitively refuted. What makes it the more valuable is that it is a form of argument of wide applicability in philosophy. It is on a quite similar principle that the hedonist says that the question of what is good morals and what bad must in the last resort come down to a question of feeling pleasure or pain. For, he urges, whatever we desire we take satisfaction in; and if we did not take satisfaction in it, we should not desire it. Thus the only thing we ever can desire is gratification, or pleasure; and all deliberate action must be performed with a view to enjoyment. So too every idealist sets out with an analogous argument, although he may probably shift his ground insensibly as he proceeds further. But at first he will say: When I perceive anything I am conscious; and when I am conscious of anything I am immediately conscious of something, and it is through that immediate consciousness that I become conscious of whatever is in my consciousness. Consequently, all that I learn from perception is that I have a feeling, together with whatever I infer from that as a premiss.

A single answer will suffice for all such argumentation. What they all assume to be necessary is, on the contrary, impossible. No desire can possibly desire its own gratification; no judgment can judge itself to be true; no reasoning can conclude itself to be sound. For the contrary positions stand on one ground and must stand or fall together, so that to refute one is to refute all. Take, then, the question of whether a judgment judges itself to be true. Unquestionably if one judgment does so, every judgment does so. I myself formerly gave something like the following argument to prove that every judgment does so. Consider the proposition:

<div style="text-align:center">This proposition is not true.</div>

This is certainly a proposition. Hence by the definition of a proposition it is either true or not. But suppose it involves no falsity. Then it will follow that it is not true and does involve falsity, thus reducing that hypothesis to absurdity. The proposition is therefore not only false but absurd and self-contradictory. But all that it explicitly asserts is that it is not true. There is certainly no contradiction in saying that it is not true: it is the very conclusion we have come to. Consequently, the only way in which it can involve contradiction is by expressing at the same time the assertion that it is true. We must therefore conclude that every proposition, in the very propositional form itself, expresses the assertion of its own truth.

That may sound forcible, but it is a huge *petitio principii.* For if no proposition asserts its own truth, none asserts its own falsity; and since this asserts nothing else, it would under that hypothesis assert nothing at all and therefore not be a proposition. And if it is not a proposition the whole argument falls to the ground. Now hear an argument on the other side of the question. If a proposition asserts its own truth, it asserts something about itself; and indeed, manifestly the whole question is whether or not a proposition does assert anything about itself. If it asserts anything about itself and about its assertion, it certainly asserts that it asserts what it does assert. But if that be the case, these two propositions

It rains.

I assert that it rains.

are one and the same proposition, or if not, the second forms a part of what the first asserts. But now consider the precise denials of the two. They are

It does not rain.

I do not assert that it rains.

Manifestly the second denial asserts much less than the other. Consequently the proposition which it denies asserts much more than the other. It appears, therefore, that the proposition "It rains" does not itself assert that I assert it rains; but when I utter the proposition "It rains" I afford you the evidence of your senses that I assert it rains. This appears to me unanswerable, and it was this argument which called my attention to the fallacy of my former reasoning.

Suppose two witnesses *A* and *B* to have been examined, but by the law of evidence almost their whole testimony has been struck out except only this:

A testifies that *B*'s testimony is true.

B testifies that *A*'s testimony is false.

Common sense would certainly declare that nothing whatever was testified to. But I cannot admit that judgments of common sense should have the slightest weight in scientific logic, whose duty it is to criticize common sense and correct it.

But I have another argument of a pragmatistic kind. Although Aristotle defines a proposition as a symbol that is either true or false,[15] that is not properly the definition of it. However, waiving the question of propriety, I have a right to use the term *proposition* for the nonce in the sense of a symbol which separately indicates its object. Then the principle that every proposition is either true or false becomes either an *axiom* or a *theorem*. But we cannot admit axioms in these days. Why then should the principle be accepted? To say that every proposition is either true or false is to say that whatever the predicate, X, of a proposition may be, its subject S is either X or not X. But this is the principle of Excluded Middle, and the principle of excluded middle, as we saw in the last lecture, merely defines individuality.[16] That is, to say that the principle of excluded middle applies to S is no more than to say that S, the subject of the proposition, is an individual. But how can that be? We know very well that universal propositions have general subjects of which the principle of excluded middle is /not/ true. That is, it is not true that "all men are either tall or not tall." The logic of relatives furnishes the solution, by showing that propositions usually have several subjects, that one of these subjects is the so-called Universe of Discourse, that as a general rule a proposition refers to several Universes of Discourse, the chief of which are Singulars, and that all propositions whatsoever refer to one common universe,—the Universal Universe or aggregate of all Singulars, which in ordinary language we denominate the Truth. The analysis of the logic of relations shows that such is the fact, and by the aid of the categories we can easily see why it should be so. A proposition is a symbol which separately INDICATES its object, and the representation in the proposition of that object is called the *subject* of the proposition. Now to INDICATE is to represent in the manner in which an index represents. But an index is a representamen which is such by virtue of standing in a genuine reaction with its object; while a singular is nothing but a genuine reacting object. It does not follow that the subject of a proposition must literally be an index, although it *indicates* the object of the representamen in a manner like the mode of representation of an index. It may be a precept by following which a singular could be found. Take for example the proposition:

Some woman is adored by every Catholic.

This means that a well-disposed person with sufficient means could find an index whose object should be a woman such that allowing an ill-disposed person to select an index whose object should be a Catholic, that Catholic would adore that woman.

Thus the subject of a proposition if not an index is a precept prescribing the conditions under which an index is to be had.

Consequently, though the subject need not be individual, the object to which the subject of a proposition applies must be the object of a possible index and as such it must be such as it is independently of any representamen or other Third. That is to say it must be *real*.

Consequently, it is impossible that a proposition should relate to itself as its object, since as long as it has not yet been enunciated it possesses characters which are not independent of how they may be represented to be.

It is, therefore, quite impossible that a proposition should assert its own truth, or what comes to the same thing, that a desire should desire its own gratification, or that an argument should conclude its own cogency, excepting only in that sense in which a point may map itself to itself, namely, as a special case under a general representation.

Consequently, when Sigwart tells me that in reasoning about a logical criterion I have to *rely* upon a feeling of logicality, he puts the cart before the horse in an utterly impossible way. He supposes that I first *feel* that a certain inference *would* gratify my sense of logicality and then proceed to draw it. But I beg to tell him that in no case whatever is it possible to *feel* what *would* happen. We *reason* about what *would* happen and we feel what *has* happened. We first draw the inference and having drawn it, if we turn our attention to our feelings we become cognizant of a sense of satisfaction. But when we have drawn the inference we have already believed in it and are satisfied, and if we become aware that the inference gives us pleasure, that is a subsequent experience upon contemplating what has happened, and it does not so much as furnish a good reason for renewed confidence in the inference, except so far as the feeling may be a sign that we should draw the same inference every time, and that it is not a mere aberration of mind.

Logic is the criticism of conscious thought, altogether analogous to moral self-control; and just as self-control never can be absolute but always must leave something uncontrolled and unchecked to act by primary impulse, so logical criticism never can be absolute but always must leave something uncriticized and unchecked. But to argue from this that logical criticism is mere feeling, would be like arguing in the other case that the only ground of morality is mere impulse.

Besides, if Sigwart's reasoning is good for anything at all, it goes to prove what he in fact deduces from it and founds his logic upon, namely that sound logic *consists* in the gratification of a feeling, which not only amounts to denying the distinction of truth and falsehood but would necessitate the admission that because my refutation of his position is entirely satisfactory to me and titillates my logical *Gefühl* in the most agreeable manner, therefore this refutation is, *ipso facto*, sound logic.

Were the Holy Father in Rome to take it into his head to use his Infallibility to command the Faithful, under pain of excommunication, to believe everything that any Protestant ever had said or ever should say, he would put himself into a position very much like that *[which]* Sigwart assumes in reducing logicality to a Quality of Feeling.

IV

The irreducibility of the idea of Thirdness appears to me to be evidently proved in the Logic of Relations. Yet Mr. A. B. Kempe, formerly president of

the London Mathematical Society, who has made an important contribution to a part of the Logic of Relations in his "Memoir on the Theory of Mathematical Forms" in the *Philosophical Transactions* for 1886,[17] plainly does not share my opinion and without directly mentioning me calls attention to certain phenomena whose interest to his mind evidently is that he regards them as refuting the irreducibility of Thirdness. This objection springing, as it does, from exact analysis, should command my most serious consideration.

In order to expound Mr. Kempe's opinion I must define a few technical terms. In ordinary logical analysis such as is required in the algebraical or other purely formal treatment, it is sufficient to consider Category the Second as a two-sided element in the phenomenon, a *Reaction*, involving two objects which are differently related to one another, but having no general distinctive characters. In like manner Category the Third in the same analysis is regarded as a triadic element of the phenomenon without there being any reason for putting one of the triad of singulars which may be concerned in it as the First, rather than either of the others, nor any one as specially Second or Third. There are other purposes, however, for which it is necessary to conceive that in a reaction the first object [is distinguished] from the second by a general character common to all *firsts*, all *seconds* having their general character; and similarly in all triadic facts distinctive general characters are to be attributed to the First, the Second, and the Third of the three objects concerned. If two singulars A and B react upon one another, the action of A upon B and the action of B upon A are absolutely the same element of the phenomenon. Nevertheless, ordinary language makes the distinction of *agent* and *patient*, which, indeed, in the languages that are familiar to us is given great prominence; and this is the case with the majority of the languages of all families, as well as the Procrustean bed imposed by the grammarians allows us to make out their real character. But in all families, languages are found in which little or nothing is made of the distinction. In Gaelic, for example, the usual form of expression places what we should call the subject in an oblique case,—the genitive, in that language, but in some languages it is rather an ablative or an instrumental case. This distinction of agent and patient is sometimes useful even in philosophy. That is, a formal distinction is drawn between the action of A on B and the action of B on A although they are really the same fact. In the action of A on B, the patient B is conceived to be affected by A while the agent A is unaffected by B. A is modified in the action so far as to be in an active state; but this is conceived to be a certain Quality that the agent takes on during the action in which Quality the patient in no way participates, while the patient, on the other hand, takes on a relative character which can neither exist nor be conceived to exist except as correlative to an agent. That is the distinction of agent and patient. So in a triadic fact, say, for example

$$A \text{ gives } B \text{ to } C$$

we make no distinction in the ordinary logic of relations between the *subject nominative*, the *direct object*, and the *indirect object*. We say that the proposition

has three *logical subjects*. We regard it as a mere affair of English grammar that there are six ways of expressing this:

A gives *B* to *C*	*A* benefits *C* with *B*
B enriches *C* at expense of *A*	*C* receives *B* from *A*
C thanks *A* for *B*	*B* leaves *A* for *C*

These six sentences express one and the same indivisible phenomenon. Nevertheless, just as *[in]* conceiving of two reacting objects we may introduce the metaphysical distinction of *agent* and *patient*, so we may metaphysically distinguish the functions of the three objects denoted by the *subject nominative*, the *direct object*, and the *indirect object*. The *subject nominative* denotes that one of the three objects which in the triadic fact merely assumes a non-relative character of activity. The *direct object* is that object which in the triadic fact receives a character relative to that agent, being the *patient* of its action, while the *indirect object* receives a character which can neither exist nor be conceived to exist without the cooperation of the other two. When I call Category the Third the Category of Representation in which there is a Represented Object, a Representamen, and an Interpretant, I recognize that distinction. This mode of distinction is, indeed, *germane* to Thirdness, while it is *alien* to Secondness. That is to say, *agent* and *patient* as they are by themselves in their duality are not distinguished as agent and patient. The distinction lies in the mode of representing them in my mind, which is a Third. Thus there is an inherent Thirdness in this mode of distinction. But a *triadic* fact is in all cases an intellectual fact. Take *giving* for example. The mere transfer of an object which *A* sets down and *C* takes up does not constitute giving. There must be a transfer of *ownership* and ownership is a matter of Law, an intellectual fact. You now begin to see how the conception of representation is so peculiarly fit to typify the category of Thirdness. The object represented is supposed not to be affected by the representation. That is essential to the idea of representation. The Representamen is affected by *[the]* Object but is not otherwise modified in the operation of representation. It is either qualitatively the double of the object in the Icon, or it is a patient on which the object really acts, in the Index; or it is intellectually linked to the object in such a way as to be mentally excited by that object, in the Symbol.

It is desirable that you should understand clearly the distinction between the Genuine and the Degenerate Index. The Genuine Index represents the duality between the representamen and its object. As a whole it stands for the object; but a part or element of it represents *[it]* as being the Representamen, by being an *Icon* or analogue of the object in some way; and by virtue of that duality, it conveys information about the object. The simplest example of a genuine index would be, say, a telescopic image of a double star. This is not an *icon* simply, because an *icon* is a representamen which represents its object solely by virtue of its similarity to it, as a drawing of a triangle represents a mathematical triangle. But the mere appearance of the telescopic image of a

double star does not proclaim itself to be similar to the star itself. It is because we have set the circles of the equatorial so that the field must by physical compulsion contain the image of that star that it represents that star, and by that means we know that the image must be an icon of the star, and information is conveyed. Such is the genuine or informational index.

A Degenerate Index is a representamen which represents a single object because it is *factually* connected with it, but which conveys no information whatever. Such, for example, are the letters attached to a geometrical or other diagram. A *proper name* is substantially the same thing; for although in this case the connection of the sign with its object happens to be a purely mental association, yet that circumstance is of no importance in the functioning of the representamen. The use of letters as indices is not confined to mathematics. Lawyers particularly often discuss cases in which A contracts with B to do something. These letters are convenient substitutes for relative pronouns. A relative, demonstrative, or personal pronoun comes very near to being a mere index, if it be not accurately so. It is far more correct so to define it than to say that a pronoun is a word placed instead of a noun. It would be nearer right to say that a common noun, when subject nominative, is a word put in place of a pronoun. A degenerate index may be called a *Monstrative* Index, in contradistinction to an *Informational* or Genuine Index.

A *proposition* is a symbol which like the informational index has a special part to represent the representamen, while the whole or another special part represents the object. The part which represents the *representamen* and which excites an *icon* in the imagination, is the Predicate. The part which indicates the object or set of objects of the representamen is called the Subject or Subjects, in grammar the subject nominative, and the objects, each of which can be replaced by a Proper Name or other *Monstrative Index* without the proposition's ceasing thereby to be a proposition. How much shall be embraced in the predicate and how many subjects shall be recognized depends, for the ordinary analyses of logic, upon what mode of analysis will answer the purpose in hand. If from a proposition we strike out a part and leave its place blank, this part being such that a monstrative index being put in its place, the symbol will again become a proposition, the part which remains after such erasure will be a predicate of the kind which I call a *monad*. Here are examples:

_____ gives B to C

A gives _____ to C

A gives B to _____

If two blanks remain, I call the predicate a *dyad*. Such are

_____ gives _____ to C

_____ gives B to _____

A gives _____ to _____

If there are more than two blanks, I call the predicate a *polyad*. The entire proposition may be regarded as a predicate, the circumstances under which it is uttered, the person who utters it, and all the surroundings constituting a monstrative index which will be the subject. I term an entire proposition without a blank when it is considered as a predicate a *medad*, from μηδέv. Every proposition whatever has the *Universe of Discourse* for one of its subjects and all propositions have one Subject in common which we call the *Truth*. It is the aggregate of all realities, what the Hegelians call the *Absolute*.

Thus, to include more in the predicate than need be included is merely not to carry logical analysis as far as it might be carried: it does not affect its accuracy. But to include anything in a subject which might be separated from it and left in the predicate is a positive fault of analysis. To say for example that "All men" is the subject of the proposition "All men are mortal" is incorrect. The true analysis is that "Anything" is the subject and "____ is mortal or else not a man" is the predicate. So in "Some cat is blue-eyed" the subject is not "some cat" but "something," the predicate being "____ is a blue-eyed cat." "Something" means that sufficient knowledge would enable us to replace the "something" by a monstrative index and still keep the proposition true. "Anything" means that the interpreter of the proposition is free to replace the "anything" by such monstrative index as he will, and still the proposition will be true. Logicians confine themselves, apart /from/ monstrative indices themselves, to "Anything" and "Something," two descriptions of what monstrative index may replace the subject, the one description *vague*, the other *general*. No others are required since such subjects, "All but one," "All but two," "Almost all," "Two thirds of the occasions that present themselves in experience," and the like are capable of logical analysis.

Everybody who has studied logic is aware that the only *formal* fallacies which ordinary logic detects are confusions between "Anything" and "Something." The same thing remains true in the logic of relatives except that we now meet with fallacies owing to confusions about the order of succession of "Anything" and "Something" as if one should carelessly substitute for the proposition "Every man is born of some woman" the proposition "There is some woman of whom every man is born."

Now in mathematics as it has been developed, all such confusions are next to impossible for the reason that mathematicians confine their studies almost exclusively to hypotheses which present only systems of relationship that are perfectly regular or as nearly so as the nature of things allows. I could give you some amusing instances of confusions between some and all in actual mathematical treatises; but they are rare. Practically, the mathematician confines himself to the study of relations among sets of hypothetical singulars.

Now Mr. Kempe considers only mathematical relationships in that sense, that is, considers only relations between single objects. Moreover, he further limits himself to relationships among enumerable sets of singulars, although mathematicians are incessantly considering denumeral or endless sets, and

very often sets still more multitudinous. So confining his studies, Mr. Kempe finds that all the relationships he meets with can be represented by graphs composed of dots of various colors connected by lines. For example, the relations between ten rays of the ten-ray theorem of Optical Geometry, which von Staudt demonstrated so beautifully, and which are shown in this figure,[18]
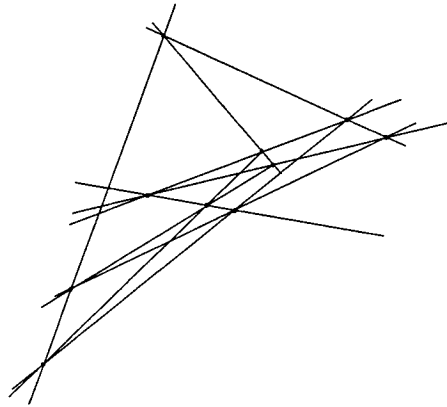

Fig. 3

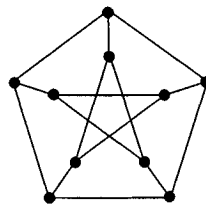are represented by Kempe in a graph substantially like [figure 4]:[19]

Fig. 4

Now Mr. Kempe seems to think that because his graphs are composed of but two kinds of elements, the spots of various colors and the lines, therefore a third category is not called for. To mathematical minds this will probably seem a formidable objection. It seemed so to me when I read it, I confess. But I will give three answers to it, each sufficient to completely reverse its force.

The first reply is that it is not true that Mr. Kempe's graphs only contain two kinds of elements. It no doubt seems at first glance that the dots of various colors represent Qualities, and the lines Reactions; and one then looks in vain for anything corresponding to Thirdness. The lines which are all drawn between pairs of points no doubt do in truth embody the Category of Duality. And in confirmation of this, it is to be remarked that the lines really dichotomize into two kinds, although Kempe draws but one kind. For *non-connection* is itself a species of connection. Thus in the graph to which I have called your attention, the pairs of lines between which there are no dots are the pairs which intersect at the ten significant points of the theorem.

But the spots, each of which ties together any number of lines and always many either of the written kind or of the unwritten, far from representing the Category of Unity, plainly embody the Category of Plurality,—the Third Category,—and it is the Surface upon which the graph is written as *one* whole which in its Unity represents the Category of Unity. So that it appears that three kinds of elements are needed for his graphs, which therefore vindicate the third category instead of refuting it as they were supposed to do.[20]

My other two answers to Kempe which would be works of supererogation must be left unperformed. I will just mention their nature. Answer number two would have consisted in the remark that Kempe does not represent in his graphs any entire system of real mathematical relationships. For example, his graph of the ten-ray theorem fails to show what triplets of rays are copunctual. This is still more strikingly the case with the nine-ray theorem, of which
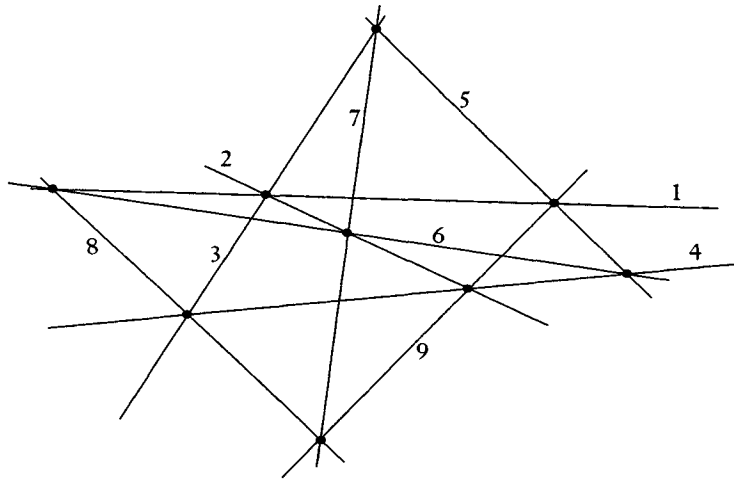

Fig. 5

[figure 5] is the geometrical figure and [figure 6] the graph drawn as Kempe has drawn [it],[21] than of the ten-ray theorem. In short, he has simply omitted from consideration all triadic relations and then points triumphantly to the fact that he has nothing to represent Thirdness, as if that proved there was no such irreducible conception. But if his graphs were modified so as to exhibit the copunctual rays and collinear points, the triadic character of his spots would come out very clearly.
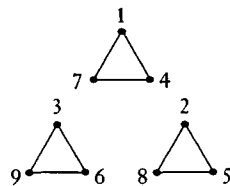

Fig. 6

Answer number three is that Kempe not only fails altogether to represent general relations, but simply gives an icon. His graphs never express *propositions*, far less *necessary consequences*. Now I invented and developed a good many years ago such a modification of Kempe's method of representation as was required to make it really express everything in mathematics. I inserted a slight sketch of it into Baldwin's *Dictionary*. It has never been published otherwise.[22] In consequence of my great interest in the working of that system, my studies of it had always followed that line and, until I came to write this lecture, it had never occurred to me to examine it in respect to its relation to the categories. On doing so, I found the three Categories copiously illustrated in the system. But what was still more interesting, a certain fault in the system, by no means of a fatal kind but still a vexatious inelegance which I had often remarked but could see no way of remedying, now, when looked upon from the point of view of the categories, appeared in a new and stronger light than ever before, showing me not only how to remedy the defect that I had seen, but opening my eyes to new possibilities of perfectionment that I had never dreamed of. I wish I could present all this to you, for it is very beautiful and interesting as well *[as]* very instructive; but it would require several lectures and lead me quite away from Pragmatism.

It is certainly hard to believe, until one is forced to the belief, that a conception so obtrusively complex as Thirdness is should be an irreducible unanalyzable conception. What, one naturally exclaims, does this man think to convince us that a conception is complex and simple, at the same time! I might answer this by drawing a distinction. It is complex in the sense that different features may be discriminated in it, but the peculiar idea of *complexity* that it contains, although it has complexity as its object, is an unanalyzable idea. Of what is the conception of *complexity* built up? Produce it by construction without using any idea which involves it if you can.

The best way of satisfying oneself whether Thirdness is elementary or not,—at least, it would be the best way for me, who had in the first place a natural aptitude for logical analysis which has been in constant training all my life long,—and I rather think it would be the best way for anybody provided he ruminates over his analysis, returns to it again and again, and criticizes it severely and sincerely, until he reaches a complete insight into the analysis,— the best way, I say, is to take the idea of representation, say the idea of the fact that the object, A, is represented in the representation, B, so as to determine the interpretation, C,—to take this idea and endeavor to state what it consists in without introducing the idea of Thirdness at all if possible, or if you find that impossible, to see what is the minimum or most degenerate form of Thirdness which will answer the purpose.

Then, having exercised yourself on that problem, take another idea in which, according to my views, Thirdness takes a more degenerate form. Try your hand at a logical analysis of the fact that A gives B to C.

Then pass to a case in which Thirdness takes a still more degenerate form, as for example the idea of "A and B." What is at once A and B involves the

idea of three variables. Putting it mathematically, it is $z = xy$ which is the equation of the simpler of the two hyperboloids, the two-sheeted one as it is called.[23]

Whoever wishes to train his logical powers will find those problems furnish capital exercize; and whoever wishes to get a just conception of the universe will find that the solutions of those problems have a more intimate connection with that conception than he could suspect in advance.

V

I have thus far been intent on repelling attacks upon the categories which should consist in maintaining that the idea of Reaction can be reduced to that of Quality of Feeling and the idea of Representation to those of Reaction and Quality of Feeling taken together. But meantime may not the enemy have stolen upon my rear, and shall I not suddenly find myself exposed to an attack which shall run as follows:

We fully admit that you have proved, until we begin to doubt it, that Secondness is not involved in Firstness nor Thirdness in Secondness and Firstness. But you have entirely failed to prove that Firstness, Secondness, and Thirdness are independent ideas for the obvious reason that it is as plain as the nose on your face that the idea of a triplet involves the idea of pairs, and the idea of a pair the idea of units. Consequently, Thirdness is the one and sole category. This is substantially the idea of Hegel; and unquestionably it contains a truth.

Not only does Thirdness suppose and involve the ideas of Secondness and Firstness, but never will it be possible to find any Secondness or Firstness in the phenomenon that is not accompanied by Thirdness.

If the Hegelians confined themselves to that position they would find a hearty friend in my doctrine.

But they do not. Hegel is possessed with the idea that the Absolute is One. Three absolutes he would regard as a ludicrous contradiction *in adjecto*. Consequently, he wishes to make out that the three categories have not their several independent and irrefutable standings in thought. *Firstness* and *Secondness* must somehow be *aufgehoben*. But it is not true. They are in no way refuted nor refutable. Thirdness it is true involves Secondness and Firstness, in a sense. That is to say, if you have the idea of Thirdness you must have had the ideas of Secondness and Firstness to build upon. But what is required for the idea of a genuine Thirdness is an independent solid Secondness and not a Secondness that is a mere corollary of an unfounded and inconceivable Thirdness; and a similar remark may be made in reference to Firstness.

Let the Universe be an evolution of Pure Reason if you will. Yet if while you are walking in the street reflecting upon how everything is the pure distillate of Reason, a man carrying a heavy pole suddenly pokes you in the small of the back, you may think there is something in the Universe that Pure Reason fails to account for; and when you look at the color *red* and ask yourself how Pure Reason could make *red* to have that utterly inexpressible and irrational

positive quality it has, you will be perhaps disposed to think that Quality and Reaction have their independent standings in the Universe.

VI

So far I have only considered whether or not the categories must be admitted as so many independent constituents of thought. In my next lecture I shall have to examine whether they all three have their place among the realities of nature and constitute all there is in nature.

I confess I wonder how any philosopher can say "Oh, *Thirdness* merely exists in thought. There is no such thing in reality." You do know I am enough of a sceptic to be unwilling to believe in the miraculous power he attributes to the mind of originating a category the like of which God could not put into the realities, and which the Divine Mind would seem not to have been able to conceive. Still, those philosophers will reply that this may be fine talk but it certainly is not argument; and I must confess that it is not. So in the next lecture the categories must be defended as realities.